An Analytical Investigation of Short-Period Flying Qualities

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Longitudinal short-period flying qualities boundaries are defined in terms of parameters of the closed-loop pilot-airplane system. The parameters are varied to match analytical boundaries to several experimental "good" flying qualities boundaries. Parameter values which provide matching are compared to values assumed good in previous analyses of a similar nature. The analytical and experimental boundaries agree well and provide a means for experimentally determining good parameter values. It is concluded that there is an upper bound on short-period frequency for good flying qualities and that pilots accept lower values of closed-loop damping ratio than those previously assumed to be acceptable.

Nomenclature

A_θ = numerator polynomial coefficient in pitch-elevator transfer function
g = acceleration due to gravity
G_c = controlled element transfer function
G_p = pilot linear transfer function
i = imaginary unit, $(-1)^{1/2}$
K_p = pilot gain
L_α = variation of lift, L, with angle of attack, α
M_i = variation of pitching moment, M, with input or motion quantity, i
q = pitching velocity
s = Laplace transform variable
$T_{\theta 2}$ = pitch angle-elevator transfer function numerator time constant
x, y = Cartesian coordinates in the complex plane, $z = x + iy$
Z_w = variation of Z-direction force with vertical velocity, w
α = angle of attack
δ_e = elevator deflection
ζ_{CL} = closed-loop damping ratio
ζ_{sp} = damping ratio of short-period oscillation
ξ, η = pole/zero cartesian coordinates in the complex plane, $z = \xi + i\eta$
θ = pitch angle
θ_i = contribution of the ith pole or zero to argument of the root-locus
τ = pilot pure time delay constant
ϕ_M = phase margin
ω_c = crossover frequency
ω_{CL} = closed-loop, undamped natural frequency
ω_{sp} = undamped natural frequency of short-period oscillation

Introduction

THE investigation of flying qualities through analysis of the closed-loop, pilot-airplane system has received much attention in recent years, e.g., Refs. 1–4. The approach used in such studies is based on the fact that, in many situations involving precision control of the airplane, the pilot may be considered to be an element in a feedback system consisting of pilot, display, control system, and airframe. If a suitable mathematical representation of pilot behavior is known, or can be assumed, the closed-loop system may then be subjected to conventional analysis procedures.

The analysis is based on the assumption that airplane flying qualities can be characterized in terms of frequency-response and closed-loop parameters of the pilot-airplane system, such as phase margin, crossover frequency, closed-loop natural frequency, and closed-loop damping ratio. If the values of these parameters which result in good pilot opinion can be specified, then the system can be analyzed so as to define airplane dynamics which will result in good flying qualities.

The implementation of such a technique has previously involved much graphical analysis to define boundaries on airplane dynamics.[1,2] It has therefore become desirable to develop a faster method of analysis to facilitate varying the system parameters. It is also desirable to employ such a method to determine, from experimental data, those values of the parameters which the pilot feels constitute good flying qualities.

In the following sections a technique is developed for analytically specifying the boundaries of airplane longitudinal short-period dynamics defined by use of the system parameters. These analytical boundaries are compared to several experimental boundaries and good values for the system parameters are deduced for each experimental situation.

Analysis

System Configuration

Analysis of closed-loop pilot control of the longitudinal short-period mode involves consideration of the pilot-airplane system shown in Fig. 1. This representation of the system is valid for situations where the pilot is concentrating on longitudinal control of the airplane attitude in response to random, or random-appearing, inputs. The effects of the flight control system and display dynamics have been neglected. While this may appear at first to be a highly restricted case, it is, in fact, representative of many situations involving precision flight path control.

The airplane short-period attitude response transfer function is[5]

$$G_c(s) = A_\theta(s + 1/T_{\theta 2})/s(s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2) \quad (1)$$

where, in terms of stability derivatives

$$\omega_{sp}^2 \doteq M_q Z_w - M_\alpha \quad (2a)$$

$$2\zeta_{sp}\omega_{sp} \doteq -(M_q + M_{\dot\alpha} + Z_w) \quad (2b)$$

$$A_\theta = M\delta_e, \; 1/T_{\theta 2} \doteq -Z_w \quad (2c)$$

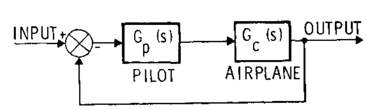

Fig. 1 Pilot-airplane system.

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The pilot transfer function used in situations involving good pilot ratings is that for the "nonequalized" pilot[6]

$$G_p(s) = K_p e^{-\tau s} \tag{3}$$

This equation is based on the assumption that the pilot considers flying qualities to be good if he is not required to supply lead or lag compensation to achieve the closed-loop and frequency-response characteristics that he desires. He simply functions as a gain constant, K_p, and a pure (involuntary) neuromuscular time delay of τ sec. The magnitude of the gain, K_p, has a definite effect on pilot ratings, but it will be assumed during this analysis that the total forward loop gain, $K_p A_\theta$, is such that K_p is always good.

The closed-loop system will be considered good by the pilot when its frequency-response and closed-loop characteristics exhibit values which he considers desirable. The system parameters which are assumed to be related to pilot opinion are: 1) phase margin, 2) crossover frequency, 3) closed-loop damping ratio, and 4) closed-loop natural frequency. Boundaries corresponding to each of these criteria will be defined analytically in the following subsections.

Phase Margin-Crossover Frequency

The phase margin and the crossover frequency of the pilot-airplane system are related through the defining equation for the phase margin:

$$\phi_M = \frac{\pi}{2} + \tan^{-1}(T_{\theta 2}\omega_c) - \tau\omega_c - \tan^{-1}\left(\frac{2\zeta_{sp}\omega_{sp}\omega_c}{\omega_{sp}^2 - \omega_c^2}\right) \tag{4}$$

If the coordinates of the short-period roots in the root-locus plane are taken to be $(\xi \pm i\eta)$, where

$$\omega_{sp}^2 = \xi^2 + \eta^2 \qquad \zeta_{sp} = -\xi/\omega_{sp} \tag{5}$$

then Eq. 4 may be rewritten, eventually, as[7]

$$(\xi - \xi_0)^2 + \eta^2 = \omega_c^2 + \xi_0^2 \tag{6}$$

where

$$\xi_0 = -\omega_c - T_{\theta 2}\omega_c^2 \cot(\phi M + \tau\omega_c)/\cot(\phi_M + \tau\omega_c) + T_{\theta 2}\omega_c \tag{7}$$

Eq. 6 is the equation of circle whose center is located at $(\xi_0, 0)$ and whose radius is

$$R^2 = \xi_0^2 + \omega_c^2 \tag{8}$$

For a specified system phase margin, ϕ_M, Eq. 6 defines a boundary of the short-period roots to provide some minimum crossover frequency, $\omega_{c_{MIN}}$. For obtaining good system response, it is normally desirable that the crossover frequency be greater than the input forcing function bandwidth; it is also assumed that the pilot desires this characteristic or a similar one. Consequently, some minimum crossover frequency should be acceptable to the pilot for a given input function.

Closed-Loop Frequency and Damping

The open-loop and closed-loop frequencies and damping ratios of the pilot-airplane system are related through the system root-locus equation

$$\arg[G(s)] = \pm 180° \tag{9}$$

Therefore any point, $(x \pm iy)$, in the root-locus plane, which lies on the root locus of the system must satisfy the condition that

$$\theta_1 + \theta_2 - \theta_3 + \theta_4 + \theta_5 = \pm 180° \tag{10}$$

where

$$\theta_1 = \tan^{-1}\left[\frac{(y+\eta)}{(x-\xi)}\right] \quad \theta_2 = \tan^{-1}\left[\frac{(y-\eta)}{(x-\xi)}\right]$$

$$\theta_3 = \tan^{-1}\left[\frac{y}{(x+1/T_{\theta 2})}\right] \quad \theta_4 = \tau y \quad \theta_5 = \tan^{-1}\left(\frac{y}{x}\right) \tag{11}$$

Specifying the point (x, y) is equivalent to requiring that the system be capable of attaining desired closed-loop properties (for some value of gain)

$$\left.\begin{aligned} \omega_{CL}^2 &= x^2 + y^2 \\ \zeta_{CL} &= -x/\omega_{CL} \end{aligned}\right\} \tag{12}$$

Equation (10) can be put into a more useful form by taking the tangent of both sides of the equation

$$\tan(\theta_1 + \theta_2 - \theta_3 + \theta_4 + \theta_5) = 0 \tag{13}$$

This eventually reduces to[7]

$$(\xi - \xi_1)^2 + \eta^2 = (\xi_1 - x)^2 + y^2 \tag{14}$$

where

$$\xi_1 = x - \left\{x + \left[\frac{xy}{(x+1/T_{\theta 2})}\right]\tan(\tau y) + \left[\frac{y^2}{(x+1/T_{\theta 2})}\right] - y\tan(\tau y)\right\} \Big/ \left\{\left[\frac{x}{(x+1/T_{\theta 2})}\right] - \left(\frac{x}{y}\right)\tan(\tau y) - \left[\frac{y}{(x+1/T_{\theta 2})}\right]\tan(\tau y) - 1\right\} \tag{15}$$

Equation (14) is the equation of a circle whose center is located at $(\xi_1, 0)$, and whose radius is

$$R^2 = (\xi_1 - x)^2 + y^2 \tag{16}$$

Equation (14) defines the locus of short-period roots for which the root-locus of the pilot-airplane system passes through the prescribed point (x, y), corresponding to the closed-loop parameters ζ_{CL}, ω_{CL}.

It should be noted that Eq. (13) is satisfied not only by the collection of possible points so that

$$\arg[G(s)] = \pm 180°$$

but also by the collection of possible points so that

$$\arg[G(s)] = 0°, 360°$$

However, it can be shown[7] that points on one side of the radius corresponding to ζ_{CL} will satisfy the 0, 360° criterion and points on the other side will satisfy the $\pm 180°$ criterion. The applicable portion of the boundary thus terminates at the radius corresponding to the value of ζ_{CL} used in Eq. (15) and (16).

Note also that Eq. (15) and (16) actually define a family of circles for constant ω_{CL} and varying ζ_{CL}. Therefore, the boundary defined by these circles is the least restrictive envelope of the circles required for establishing $\zeta_{CL} \geq \zeta_{CL_{MIN}}$.

Gain Compatibility

The effect of specifying system phase margin is to specify a system gain for each combination of longitudinal short-period frequency and damping. Thus, the possibility arises that, even though the system root-locus passes through the desired point (x, y), the gain necessary to achieve the desired phase margin may be large enough to produce undesirable closed-loop properties. The short-period poles considered good must be further restricted to include only those poles for which the gain at the desired phase margin is less than or equal to the gain at the minimum-acceptable, closed-loop damping ratio and frequency. Therefore, a further boundary in the root-locus plane is defined by

$$K\phi_M = K\zeta_{CL_{MIN}} \quad (17)$$

where

$$K\phi_M = [\,|G_p(iw)G_c(iw)|_{\phi=\phi_M}]^{-1} \quad (18)$$

$$K\zeta_{CL_{MIN}} = [\,|G_p(x+iy)G_c(x+iy)|\,\zeta_{CL} = \zeta_{CL_{MIN}}]^{-1} \quad (19)$$

and x and y have been restricted to include only acceptable values of closed-loop frequency:

$$\omega_{CL}{}^2_{MIN} \leq x^2 + y^2 \leq \omega_{CL}{}^2_{MAX} \quad (20)$$

Unfortunately it has not been possible to solve Eq. (17) in closed form. The gain-compatibility boundary must instead be located by means of an iterative solution performed on a digital computer.

Interpretation of Boundaries

Eqs. (6), (14), and (17) define a set of boundaries in the root-locus plane. A configuration whose short-period roots satisfy all these boundaries simultaneously should be considered good by the pilot—if the parameter values used to construct the boundaries are those values preferred by the pilot. A configuration whose short-period roots lie outside of one or more of the boundaries should receive a degraded rating because of the required pilot compensation and/or undesirable system characteristics. Conversely, if the short-period roots of a sufficient number of good configurations are known, then the preferred parameter values can be deduced as being those values which provide the best match of the analytical boundaries to the experimental data.

Results and Discussion

General Form of Boundaries

A typical set of boundaries is presented in Fig. 2 for a representative set of system parameters. The area enclosed by these boundaries contains only short-period roots which satisfy simultaneously all of the criteria imposed by the parameters. Variations in the values assigned to the parameters will obviously cause variations in this good area, as detailed in Ref. 7.

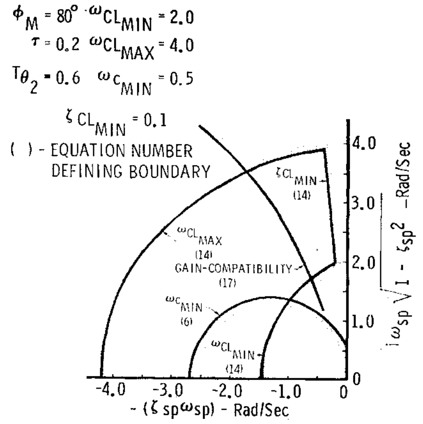

Fig. 2 Typical root-locus plane boundaries for "good" short-period flying qualities.

Determination of Good Parameter Values

Matching of experimental boundaries

In Fig. 3–5 are shown the results of attempting to match analytically defined boundaries to three experimentally derived boundaries. The "best-fit" boundaries are shown, and the parameter values used to generate these boundaries are listed in Table 1. The experimental situation defines the value of T_{θ_2} to be used, and the shape of the experimental boundary determines the required values of τ, $\omega_{CL_{MAX}}$, and $\omega_{CL_{MIN}}$. However, various combinations of $\omega_{c_{MIN}}$, $\zeta_{CL_{MIN}}$, and ϕ_M will yield essentially identical boundaries for minimum crossover frequency and gain-compatibility. Increasing the phase margin requires a lower assumed minimum crossover frequency and a higher assumed minimum closed-loop damping ratio to match a given experimental boundary. Also, McDonnell[8] indicates that pilot ratings vary in an approximately linear fashion as a function of system phase margin

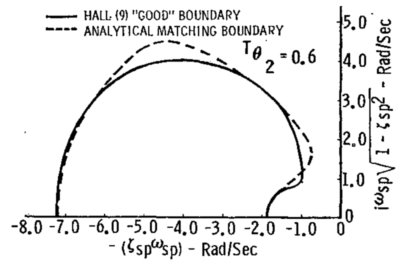

Fig. 3 Analytical match to Hall data.

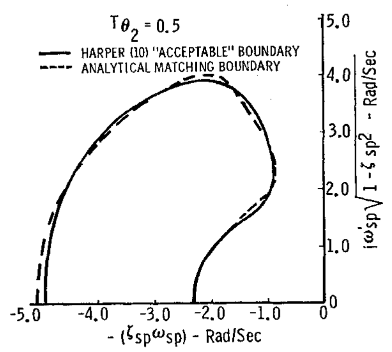

Fig. 4 Analytical match to Harper data.

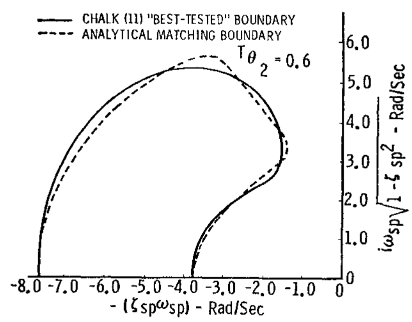

Fig. 5 Analytical match to Chalk data.

Table 1 "Best-fit" parameter values

Reference	τ (Sec)	ϕ_M (Deg)	$\omega_{C_{MIN}}$ (Rad/Sec)	$\zeta_{CL_{MIN}}$	$\omega_{CL_{MIN}}$ (Rad/Sec)	$\omega_{CL_{MAX}}$ (Rad/Sec)
Hall	0.2	80	0.25	0.16	1.7	6.1
Harper	0.2	80	0.32	0.07	2.5	4.6
Chalk	0.2	80	1.0	0.07	3.8	6.4

and that the best ratings occur at the highest phase margins. It is therefore logical that the pilot would desire as much phase margin as possible. However, for the type of system under consideration, it is difficult to consistently attain phase margins greater than about 80° at reasonable crossover frequencies. It is therefore implied that the system phase margin should be approximately 80° in order to provide good pilot ratings. Once the value of phase margin to be used is thus fixed, the associated values of $\zeta_{CL_{MIN}}$ and $\omega_{c_{MIN}}$ are determined on the basis of experimental boundaries.

Discussion of experimental data

Various difficulties were experienced in attempting to utilize the available experimental data, and the suitability of the boundaries chosen for use with the preceding analysis is open to discussion. The rating scales employed in the experimental studies were not well suited for use with a closed-loop type of analysis. The scale used in the study by Hall[9] was different from that used in the studies by Harper[10] and Chalk,[11] and in both scales more emphasis than that desirable was placed on airplane response rather than on pilot workload or compensation. Consequently, it was impossible to determine which rating boundary corresponded exactly to the case of control by an uncompensated pilot. In addition, the interpretation of the rating scale by one of the evaluation pilots participating in the study by Chalk differed from that of the other pilots. Consequently, the ratings given by that pilot had to be interpreted by the experimenter prior to use with the remaining data.

An additional factor affecting the applicability of the experimental data is the limited number of evaluation pilots employed in each of the studies. In the study by Hall two evaluation pilots were used; in the study by Harper, one; and in the study by Chalk, three pilots were employed.

The pilot ratings and their rating boundaries may also have been affected by the fact that in the evaluation procedure used in the studies by Harper and Chalk a number of open-loop type maneuvers were used, in addition to precision tracking, while the Hall data was generated during a task which involved precision tracking only. It should also be noted that the three experimental studies were all concerned with fighter-type aircraft. Consequently, the pilots may have desired somewhat more rapid response to control inputs than that desired for bomber or transport airplanes.

In spite of the difficulties experienced with use of the experimental data, the agreement in general shape between the analytical and experimental boundaries is very encouraging. The discrepancies between the boundaries could as easily be due to the effects of curve fairing in drawing the experimental boundaries as they could be due to deficiencies in the analysis.

Comparison with previous results

Table 2 contains a list of the values of the system parameters which are currently assumed to be applicable in defining good flying qualities.[1,2] The constant phase margin of 80° and the constant pilot time delay of 0.2 sec, found to provide the best matching, are seen to be consistent with previously used values. The lower minimum crossover frequencies indicated by the experimental boundaries are not necessarily in disagreement with the previously assumed value. They may be the results of lower input bandwidths in the experimental situations than the 1.0 radian/sec assumed for normal flight and used to define the $\omega_{c_{MIN}}$ value in Table 2.

Table 2 Assumed "good" parameters[1,2]

τ (Sec)	ϕ_M (Deg)	$\omega_{C_{MIN}}$ (Rad/Sec)	$\zeta_{CL_{MIN}}$	$\omega_{CL_{MIN}}$ (Rad/Sec)	$\omega_{CL_{MAX}}$ (Rad/Sec)
0.2–0.4	60–110	1.0	0.35	0.8	——

The major area of disagreement between the good parameter values found in the present study and those used in previous studies is that of the specification of closed-loop dynamics. The values of minimum closed-loop damping ratio found to provide good boundary matching were much lower than the 0.35 value currently in use. There is no ready explanation for this difference at the present time, but it may be at least partially a mission effect produced by fighter-type dynamics.

The boundary determined by use of minimum closed-loop frequency is not well defined for the three cases considered. The best matching of the boundaries is obtained if the minimum crossover frequency requirement is assumed to define the more restrictive boundary for low short-period frequencies—particularly at higher damping ratios. However, as illustrated in Fig. 2, a small part of the low-frequency portion of the experimental boundaries is best defined by use of a minimum closed-loop frequency criterion. The values used to generate the best-fit analytical boundaries are all higher than those previously assumed. This difference may also be at least partially attributable to a desire for fast response in fighter-type airplanes.

The assumption of a maximum acceptable closed-loop frequency and the resultant boundary represent a departure from previous analyses of this type. However, it should be emphasized that it was not only found necessary to assume the existence of such a frequency and boundary to provide good matching between analytical and experimental boundaries but this assumption was also validated by the degree of matching obtained. The boundary probably results from the high accelerations experienced by the pilot while he is controlling configurations exhibiting high natural frequencies.

It is of interest that the values of closed-loop total damping, $2(\zeta_{CL}\omega_{CL})_{MIN}$, which result from the best-fit parameter values are very consistent with the value which results from the assumed values listed in Table 3. This quantity is a measure of system rise time and may represent a desire, by the pilot, for some minimum response time.

Table 3 Resulting minimum closed-loop total damping

Source		Total Damping $2\zeta_{CL_{MIN}}\,\omega_{CL_{MIN}}$-Rad/Sec
Assumed "good"		0.28
"Best-fit	Hall	0.272
	Harper	0.175
	Chalk	0.266

Summary of Results

In summary, the degree of matching obtained between analytical and experimental boundaries indicates that the parameters used to characterize the closed-loop pilot-airplane system are indeed directly related to pilot opinion. It appears that the pilots rated as good configurations which exhibited much lower closed-loop damping than that previously assumed and that they required significantly higher minimum closed-loop frequencies. The shapes of the experimental boundaries also dictated the assumption of an upper bound on closed-loop frequencies for good ratings, which is a departure from the approach taken in previous closed-loop analyses.

Application and Extension

It is felt that the technique of analytical, closed-loop analysis of airplane flying qualities has great promise. Not only does the method provide further insight into those properties of the pilot-airplane system which affect pilot opinion, but it also has potential application as a preliminary design tool. Since the good flying qualities boundaries are affected by the value of the airplane-related quantity, T_{θ_2} (or L_α), as well as the static and dynamic stability terms, the method allows a relatively easy investigation of the possibility of improving flying qualities through modification of the airplane to vary these quantities. Alternately, the boundaries provide an estimation of the degree of stability augmentation needed to provide good flying qualities since the short-period dynamics actually controlled by the pilot are the augmented dynamics of the airplane.

The lack of a comprehensive set of data utilizing a suitable pilot rating scale, such as the new Cooper-Harper scale,[12] prevents the determination of a definitive set of system parameters needed to characterize good longitudinal short-period handling qualities. The development of such a set of data, combined with experimentally measured pilot transfer functions at the various test points, would greatly facilitate the development of analytically defined boundaries not only for good longitudinal short-period flying qualities but also possibly for acceptable or unacceptable flying qualities.

Conclusions

A very effective analytical technique has been developed for predicting the shapes of the boundaries on short-period roots for good flying qualities. Lack of a comprehensive set of suitable experimental data has prevented the determination of a definitive set of system parameters needed to characterize good flying qualities. However, the available experimental data indicate that pilots rate as good short-period configurations which yield lower closed-loop damping ratios and require higher closed-loop frequencies than those previously assumed. In addition, it was found necessary to assume the existence of an upper bound on closed-loop frequency, which had not been postulated in previous analyses of a similar nature. It should be noted that pilot model forms other than that assumed (i.e. with some lead or lag compensation) will provide good flying qualities. Thus, the analysis tends toward somewhat conservative results.

The analysis provides a design technique for easily evaluating the short-period flying qualities of an airplane. Alternatively, the technique can be used to indicate design changes necessary to provide good pilot ratings for given short-period dynamics.

It is anticipated that a suitable body of experimental data would permit the technique to be expanded to predict flying qualities boundaries for pilot opinions other than good.

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